THE EMPIRE STATE BUILDING

EDUCATIONAL MATERIALS

MATH: GRADES 6-8

LANDMARKS THROUGH THE VIEWFINDER: UNDERSTANDING & CALCULATING THE DISTANCE TO THE HORIZON

Exhibit Connections: Stephen Wiltshire, Scenes of NYC; Observation Deck

OVERVIEW OF LESSON PLAN

This middle school math lesson consists of a series of tasks that are designed around the Empire State Building's viewfinder exhibit and art exhibit and will investigate concepts related to the "horizon." When we are looking at important landmarks in the Empire State Building exhibit viewfinders, how far away are the sights that we see? When Stephen Wiltshire created his artwork, how does he show distance and buildings disappearing as they reach the horizon?

We will ask students to consider: When we're standing at the beach, what is the "line" that divides the ocean and the sky? How far away is it? How far can we see before the earth disappears and meets the sky? Why is the distance of our sight limited? How does the curvature of the earth determine how far we can see? Consider: If the world were flat, the horizon would be infinitely far away at the end of the earth, and we would be able to see forever!

In this series of activities, we will prepare students to look out from the Observation Deck at the Empire State Building and use the viewfinders to consider how the height of the building impacts what they will see. Specifically, students will investigate **how the height of the building affects the distance we can see to the horizon.** This will require students to use math that is reflected in numerous state standards: representing sight lines and real-life objects in two-dimensional drawings; measurement and using scale models; using algebraic equations, substitution for variables, order of operations, and square roots; using the Pythagorean Theorem; and using all these skills to conduct research about maps and the landmarks near the Empire State Building.

The Empire State Building has one of the world's highest and most famous observation points in the entire world. How far can we see when standing at the top? How far can we see when we are standing at the beach looking out at the ocean? Why is the distance different? How far can we see from the top of a mountain? Or if we were in the International Space Station?

This lesson is designed to incorporate math into students' appreciation of the exhibits and the views at the Empire State Building and to allow teachers to customize the materials for the level of their students. The pre-trip lesson has several components that incorporate a wide range of math concepts appropriate for grades 6 to 8, and all focus on concepts related to distance and the curvature of the earth.

OBJECTIVES

Students will deepen their understanding of a range of middle school math standards, including:

- Understanding and using scale drawings, measurement, and scale
- Representing 3-dimensional shapes
- Substitution into algebraic equations, order of operations, square roots
- Pythagorean Theorem (adjustable to suit their grade level)
- All of the math content will be addressed in real-life scenarios related to heights of buildings and the curvature of the earth, and are designed to link to the exhibits with the art and viewfinders that consider the landmarks we can see from the observation deck.

SUGGESTED TIME ALLOWANCE: One hour

RESOURCES/ MATERIALS:

- Ruler
- Provided handouts and worksheets

ACTIVITIES/ PROCEDURES:

PRE-TRIP ACTIVITY
WRITING/HYPOTHESIZING ABOUT THE HORIZON

Students will begin to preview their trip by hypothesizing and brainstorming about the "horizon" and why it exists. The teacher can lead a discussion of thought-provoking questions and ask students to estimate how far away the horizon is when they look from various locations. These topics could also be addressed with a series of writing prompts. Students will predict whether they expect the sight distance to be farther from the top of the Empire State Building and explain why they think so. The key goal is to engage students in the excitement of their upcoming trip to the Empire State Building, imagine what they might be able to see from the top, and develop an understanding of the way the curvature of the earth impacts what we see. Teachers should encourage students to consider how the height of their viewpoint might change the angle at which they observe.

DRAWING MODELS TO PROVE THAT HEIGHT AFFECTS DISTANCE TO HORIZON

Using the handout provided worksheet, students will draw buildings of different sizes on a curved shape representing the surface of the Earth to see if they can prove or disprove their hypotheses from this writing prompts: Does a higher building allow a longer sight distance to the horizon? Using a ruler, they will draw the "sight line" from the top of the building to the farthest point they would see—the farthest point in a straight line that touches the surface of the earth. Next, they will measure and label in their diagram the height of the building and the distance to the horizon.

After drawing four different-sized buildings, they will write and discuss their conclusions: Does the view from a higher building allow them to see farther around the curvature of the earth? (It does.) And, at higher math levels, students can consider whether the relationship between the height and sight distance is a linear relationship. That is, if we double the height of the building, does that double the distance to the horizon? (It does not.) In fact, they might see that the function is quadratic (involving a square root), because a building four times higher creates a horizontal sight distance twice as far; and a building nine times higher creates a horizontal sight distance three times as far. This activity involves substantial three-dimensional thinking and representation, as well as careful measurement. It also includes the use of the scientific method, asking students to hypothesize and then prove or disprove their hypothesis, and includes calculating ratios and proportions.

USING AN ALGEBRAIC FORMULA TO CALCULATE THE DISTANCE TO THE HORIZON

This task will allow students to use a formula (which is provided to them) to calculate the distance to the horizon when given the height of the viewpoint. It is ideal for grades 6 to 8, because it allows students to substitute values into an equation to find an answer and it requires the use of order of operations and square roots. Students will use the formula to calculate the distance to the horizon from a variety of locations, including the Observation Deck of the Empire State Building. **However, if students are at a higher grade level and can use the Pythagorean Theorem to calculate the distance themselves, teachers could skip Worksheet #3 and replace it with Worksheet #4.**

> **A brief note for teachers about the math:** If students complete the calculations correctly using the formula, they will find that the distance from the top of the Empire State Building to

the visible horizon is approximately 40 miles in each direction. However, we should note that this is the distance to an object located at ground level (for example, a lake, a highway, or a small building). But if students look from the Observation Deck at another object at a significant height (such as a mountain or another high building), they will be able to see landmarks that are significantly farther than 40 miles away. For example, if we looked from the Empire State Building at a mountain that is 1,500 feet tall (similar in height to ESB), its top would be visible approximately 80 miles away (twice as far as the horizon).

USING THE PYTHAGOREAN THEOREM TO CALCULATE THE DISTANCE TO THE HORIZON
In this activity (ideal for grade 8), students will figure out how to represent three points in a drawing—the top of a building, the horizon, and the center of the earth. Those three points form a right triangle. Students can conduct research to find the radius of the earth and the height of the Empire State Building (or other objects, such as Mount Everest or the International Space Station). This will allow them to use the Pythagorean Theorem to find the missing length (the distance to the horizon). This activity is a challenging application of the Pythagorean Theorem and could replace Worksheet #3 for some grade levels. Alternatively, teachers might choose to do Worksheet# 4 first, and then provide the formula in Worksheet #3 for students to confirm their answers.

RESEARCHING ON GOOGLE MAPS TO PREDICT/IDENTIFY SITES VISIBLE FROM THE EMPIRE STATE BUILDING
In the final pre-trip activity, students will use the information they discovered in the math tasks (figuring out the distance they will be able to see from the Empire State Building to the horizon) to investigate on a map what they expect to see from the Observation Deck. They will use a map with the Empire State Building in the center and, using its scale, identify landmarks within the field of view that they expect to see. They will research using Google Maps to find large/famous landmarks (or mountains or towns) that they hope to see when they go on their trip. They will identify a series of landmarks (10, 20, 30 and 40 miles away) in various directions (north, south, east, west). They will make a list of the landmarks that they identified (including the direction and distance from the Empire State Building and bring it with them on their trip.

ON-SITE ACTIVITIES One hour

EXHIBIT: STEPHEN WILTSHIRE (15-20 minutes)
What do students notice about Stephen Wiltshire's artwork? Can you find the landmarks you identified on Google Maps? How does Wiltshire use perspective to show distance?

EXHIBIT: SCENES OF NYC (15-20 minutes)
Use three of the viewfinders to locate famous landmarks in New York City, at least one that you want to locate from the Observation Deck. What do you notice about the view from the viewfinder? How do you think your height and distance from the Observation Deck will affect your view of the landmark?

EXHIBIT: OBSERVATION DECK (15-20 minutes)

While on the observation deck, look at your map to identify the landmarks you expected to be able to see. Can you find the landmarks that are 10 and 20 miles away? Can you see the landmarks that are 30 or 40 miles away? Depending on the size of the objects, the air quality, and obstructions blocking your view, some landmarks may not be visible. However, you should be able to confirm the accuracy of your research because the landmarks 30 and 40 miles away will be near the horizon and difficult to see clearly. How does your view compare to Stephen Wiltshire's artwork? How does it compare to the view from the viewfinder?

POST-TRIP ACTIVITY

Students will write a reflection on their trip. They will explain what parts of their research were successful, which parts were inaccurate, and try to explain anything that was not successful.

EVALUATION AND ASSESSMENT

Teacher can review students' work in worksheet #1-7 to assess their understanding of how to represent three-dimensional objects on a two-dimensional drawing; the use of the drawings to represent the curvature of the earth and the horizon; their use of the formula to calculate the distance to the horizon, including algebraic substitution; and the use of the Pythagorean Theorem to calculate the distance to the horizon.

PRE-TRIP WORKSHEET

PROVE THAT HEIGHT AFFECTS DISTANCE TO HORIZON WRITING/HYPOTHESIZING ABOUT THE HORIZON

- When you stand at the beach and look out at the horizon, how far do you think you can see?
- If you were on the 3rd floor of a building next to the beach, would you be able to see farther? Why do you think so?
- How far do you think you would be able to see if you were standing on the top of Mount Everest? Or if you were riding in the International Space Station? Why?
- Would it be different if the Earth were flat?
  Imagine if you were on top of Mount Everest and the Earth were flat.
  How far would you be able to see? Why?

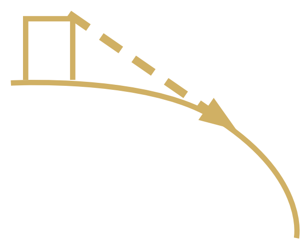
PRE-TRIP WORKSHEET

NAME______________________________________ **DATE**______________________

PROVE THAT HEIGHT AFFECTS DISTANCE TO HORIZON

Instructions: Each of the curves below represents the earth. (It is not drawn to scale.) Draw a building on the top of each of the four curves to figure out whether a taller building allows us to see a farther distance to the horizon. Use a ruler to measure the height of each building you draw, and the distance (in a straight line) to the farthest point you would be able to see. Label your diagrams. Look at the example on the right—and make sure you draw four different buildings, all of different heights. When you finish, write a conclusion: What is the relationship between the height of the building and the distance to the horizon?

Example:



PRE-TRIP WORKSHEET

NAME____________________________________ **DATE**____________________

USE A FORMULA/ALGEBRAIC FUNCTION TO CALCULATE THE DISTANCE TO THE HORIZON

Mathematicians have figured out that the relationship between the height and the distance to the horizon can be represented in an algebraic function. The function is nonlinear and includes a square root—and you can use it to figure out the distance you can see from any height.

Formula

$d = 1.17\sqrt{H}$

where d = distance to the horizon (in miles)
and h = height of the viewpoint (in feet)

PROBLEMS

Use the formula and substitute values to find the answers to these questions:

1. If your eyes are five feet above the ground, calculate the distance you can see to the horizon when you are standing on the beach.
2. If a lifeguard at the beach is sitting in a tall wooden chair, and her eyes are 16 feet from the ground, how far can she see to the horizon?
3. Research the height of the Empire State Building Observation Deck and calculate the distance you will see when you trip.
4. Research the height of Mount Everest. How far will you be able to see when you hike to the top?
5. What floor do you live on at home (or what floor are you on at school)?
   Estimate how high your window is from the ground. If you look out the window and imagine if there were no buildings or trees blocking your view—use the formula to figure out how far you would be able to see.

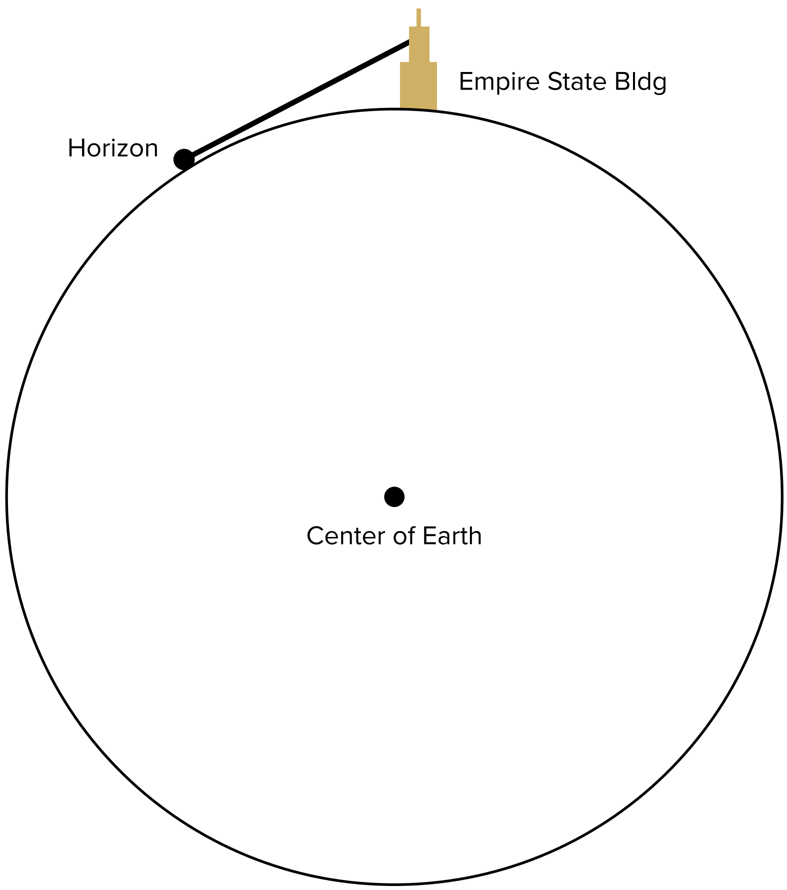
PRE-TRIP WORKSHEET

NAME____________________________________ **DATE**____________________

USE THE PYTHAGOREAN THEOREM TO FIND THE DISTANCE TO THE HORIZON FROM A HIGH VIEWPOINT

Instructions: Use the diagram below (which is not to scale) to figure out the distance from the top of the Empire State Building to the farthest point you can see at the horizon. Can you figure out how to draw a right triangle using the three points, and then find the lengths of the sides? You may need to do research to find the radius of the earth and the height of the Empire State Building. (Remember to convert the height of the building into miles if that is the unit you are using for the earth and distance.)

After you find the distance from the Empire State Building to the horizon, try other heights! How far would you see from the top of Mount Everest? From the International Space Station?



PRE-TRIP WORKSHEET

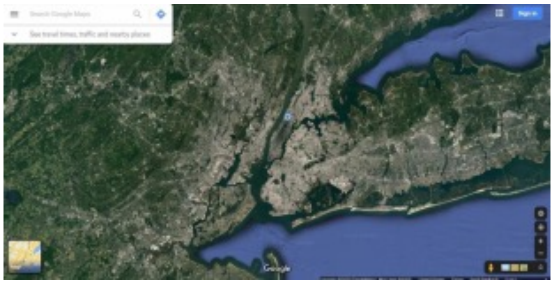
NAME_______________________________________ **DATE**____________________

PREDICT/IDENTIFY SITES VISIBLE FROM THE EMPIRE STATE BUILDING

Using your information from the previous questions above, find a map online (such as Google Maps) and use the scale to figure out everything you will be able to see from the top of the Empire State Building. For example, put the Empire State Building in the center of your map and then find the distance you can see in every direction.

After you have found a map that includes the area you will be able to see from the Observation Deck of the Empire State Building, choose some landmarks that you expect to be able to see in each direction. Choose large and famous sites, such as bridges, stadiums, other large buildings, mountains, lakes, or anything else you think you will be able to see from far away.

Make a list of 10 landmarks that you will look for when you are on your trip. Choose a few landmarks in each direction (north, south, east, and west) and a few landmarks that are approximately 5 miles away, a few 10 miles away, and as far as you expect to see based on your math calculations.



Create your list in the table on the Empire State Building Trip Note-Taking Sheet and bring it with you on your trip. Before your trip, complete the first three columns of the chart —Identifying the 10 landmarks and the direction and distance you expect to see them.

On your trip, observe carefully in the art exhibition of Stephen Wiltshire's sketches.
Look carefully at his incredible drawings and try to see if they include any of the landmarks you identified in your research.

- How far away are the buildings and sites in Wiltshire's art?
- How does the artist make some items appear to be farther away?
- Can you see the horizon in his artwork?

Next, look carefully from the Scenes of NYC exhibit viewfinders to identify famous landmarks.

- Do you see any of the items from your list?
- Does the exhibit provide information about the distance from the Empire State Building to the famous landmarks?
- Does it confirm or contradict any of your predictions?

Walk around the Observation Deck and look in all directions.
Find as many of the landmarks on your list as you can. Which are visible? Which are not?

Finally, take note of other landmarks that you can see that are as far as you can see toward the horizon.

- Can you see farther than you expected? Or less distant than you expected?
- How does the trip compare to your predictions?

TRIP WORKSHEET

NAME____ DATE_____________________

EMPIRE STATE BUILDING TRIP NOTE-TAKING SHEET

NAME OF LANDMARK	DISTANCE FROM ESB	DIRECTION	Can you see this landmark in the Stephen Wiltshire drawings? in the Viewfinders Exhibit? from the Observation Deck? Take notes.

POST-TRIP WORKSHEET

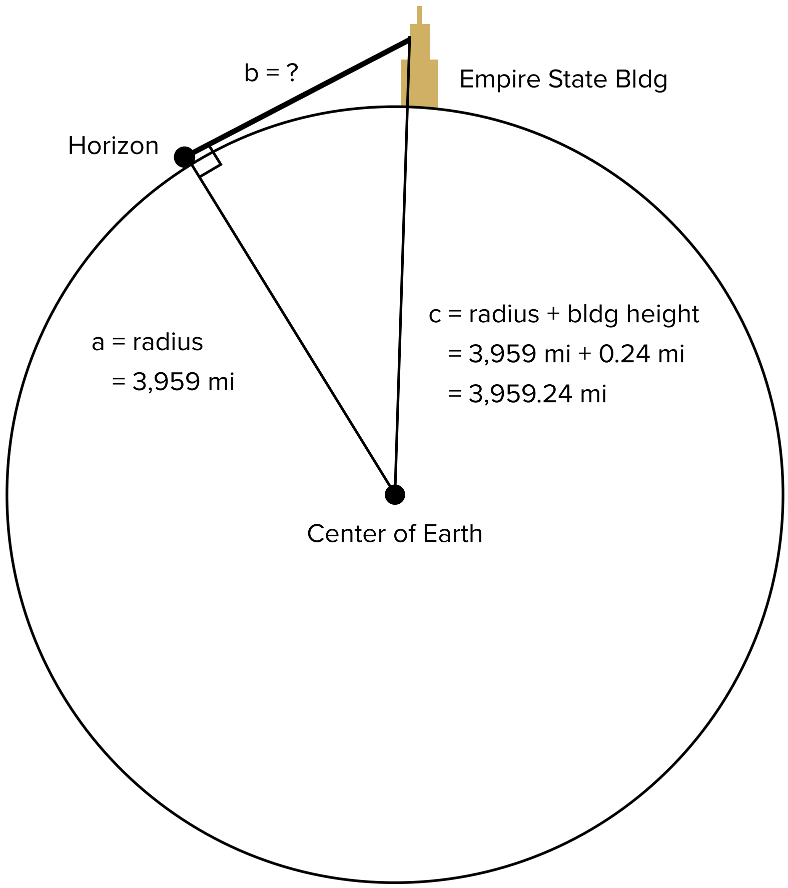
NAME______________________________________ **DATE**______________________

POST-TRIP WRITING REFLECTION QUESTIONS

1. Discuss a few landmarks that you saw in Stephen Wiltshire's art exhibit. Explain how the artist represented landmarks that are closer and farther away.
2. Discuss some landmarks that you expected to see from the Observation Deck. Were they visible where you expected them to be? If not, why do you think your prediction was inaccurate?
3. What did you see on the horizon? Do you think the distance to the horizon matched your prediction? Why or why not?

TEACHER ANSWER KEY/DATA

Using the Pythagorean Theorem to Find the Distance to the Horizon from a High Viewpoint



PYTHAGOREAN THEOREM SOLUTION

$a^2 + b^2 = c^2$

$3{,}959^2 + b^2 = (3{,}959 + 0.24)^2$

$^2 = 1$

b 900

b = 43.6 miles (using 1250 ft as height of building, or approximately 0.24 miles)